Statement of Inquiry/Problem or Issue:
Educational research has demonstrated that students learn best when actively involved in constructing their own learning. The problem is the lack of published materials that will engage students to reason and understand how mathematics works and how it can be applied. Math topics need to be presented in the context of real world situations whenever possible. Since these real world situations are not found in current textbooks, I have completed the necessary research to develop and publish teaching/learning materials for the courses MAT021 Basic Algebra I and MAT022 Basic Algebra II. These curriculum materials are in the form of 2 to 4 page handouts that can be used in a classroom environment where collaborative learning is the norm. The handouts are available in a 3-ring binder that will be left in the Office of Professional Development. I encourage Math faculty to copy and use these handouts as group activities for their classroom. Please feel free to contact me about how to best implement these activities into your classes.

Many instructors do not try small group learning because there is little published material that lends itself to such instruction. I have tried to develop a series of handouts that help students to learn mathematical concepts, attain a higher level of quantitative literacy, and achieve their academic and career goals. These curriculum materials go beyond the teaching of algebraic skills. Although the skills of Algebra are an important component of the curriculum materials, the main focus is on the study of patterns in data, functional relationships, and the underlying structure of the subject. Students discover the concepts and applications of the subject by investigating problem situations that make algebra meaningful to students. Algebra is explored as a way to understand and solve problems in the world around us. In particular, important connections are made between functions and variables and equations by actively engaging students in constructing their own learning.

Goal Statement:
The goal has been to develop a series of handouts for MAT021 Basic Algebra I and MAT022 Basic Algebra II that help students obtain a deep understanding of the course content and how it applies to courses in other disciplines, careers in almost every field, effective citizenship in understanding public policy, being an informed consumer, interpreting economic trends, evaluating health risks, and sound financial planning.

Rationale for the Inquiry:
Retention of developmental students has been a constant problem for most two-year colleges and Northern Essex Community College is no exception. To increase retention in mathematics we must use a variety of instructional strategies that improve student learning. Research suggests that we use active learning techniques. The materials I have developed give students the opportunity to be actively involved in the learning of mathematics. These handouts engage our students in meaningful activities that promote quantitative literacy.

Summary:
In conclusion, I believe that the new discovery based curriculum materials that I have produced will enhance our student's conceptual understanding and provide real life applications of developmental algebra. Use of this material in an active learning environment should increase student success in developmental mathematics and beyond. Attached is a short sample of these curriculum materials.

Discovery Adventure 1: From Arithmetic to Algebra

Suppose you are given two problems to solve:

1. Acme Auto Rental advertises that a compact car can be rented for $19.95 per day plus 12 cents for each mile driven. Suppose you rent one of these cars for a day at a total cost of $25.95. How many miles did you drive?

2. Solve the following equation for m: $19.95 + 0.12 \cdot m = 25.95$.

a. Explain the similarities and differences in these two problems.

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b. Describe the unknown quantity that you are looking for in problem 1.

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c. Describe the known quantities that you are given in problem 1.

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d. Show the calculations you need to complete to answer the question in problem 1.

e. If you let the letter m represent the unknown quantity in problem 1, explain how the equation in problem 2 can represent the situation in problem 1.

f. Assuming the equation in problem 2 is a model for the situation in problem 1, describe the two steps you need to complete to solve for m.

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Feedback

The equation in problem 2, $19.95 + 0.12 \cdot m = 25.95$ is a symbolic model of the situation in problem 1 because it uses the known and unknown values to construct a true statement showing the process of how you pay for the rental car. In this situation the unknown quantity was represented by the variable, *m*, while the known quantities were the \$19.95 price per day, the \$0.12 per mile, and the \$25.95 total cost.

The following two calculations were needed to find the solution to problem 1.

$$\begin{aligned} \text{Cost for mileage} &= \text{Total cost} - \text{ Fixed cost per day} \\ &= 25.95 - 19.95 \\ &= \$6 \end{aligned}$$

$$\begin{aligned} \text{Distance driven} &= \frac{\text{Cost of mileage (\$)}}{\text{Cost per mile}} \\ &= \frac{6}{0.12} \\ &= 50 \text{ miles} \end{aligned}$$

The steps in solving the equation, $19.95 + 0.12 \cdot m = 25.95$ for *m* involves the same two calculations as above. However, we will show how to apply those steps using two basic properties that can be used in solving any conditional equation.

- Add or subtract both sides of an equation by any constant or variable term.
- Multiply or divide both sides of an equation by any nonzero constant or variable term.

$$\begin{aligned} 19.95 + 0.12m &= 25.95 \\ 19.95 + 0.12m - 19.95 &= 25.95 - 19.95 \\ 0.12m &= 6 \\ \frac{\cancel{0.12}^{1} m}{\cancel{0.12}_{1}} &= \frac{6}{0.12} \\ m &= 50 \end{aligned}$$

Discovery Adventure 2: Body Fuel

To determine a person's energy or calorie needs, you need to find the Basal Energy Expenditure (*BEE*), which shows the minimum amount of energy needed by the body for basic life processes such as breathing, body temperature, gland functioning, heart activity, and circulation. The *BEE* is measured in calories using formulas for men and women that are based on weight, height, and age. For example, using the variables W for weight in kilograms (kg), H for height in centimeters (cm), and A for age in years, we have

Men: $BEE = 66 + (13.7 \cdot W) + (5 \cdot H) - (6.8 \cdot A)$

Women: $BEE = 655 + (9.6 \cdot W) + (1.7 \cdot H) - (4.7 \cdot A)$

a. Ask a friend to volunteer his/her weight, height, and age and record the results below.

Weight: Height: Age:

b. What type of unit conversions must be completed before applying the formula?

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c. Use the fact that 1 kg = 2.2 lbs to convert your friend's weight from pounds to kilograms. Round your answer to the hundredths place.

d. Use the following facts, 1 ft = 12 in. and 1 in. = 2.54 cm, to convert your friend's height from feet and inches to total inches and then to centimeters. Round your answer to the hundredths place.

e. Substitute your friend's weight, height, and age into the appropriate *BEE* formula and calculate the number of calories needed to sustain your friend's life while at rest. Round to the nearest calorie.

f. The male *BEE* formula contains 4 terms: 66, $13.7W$, $5H$, and $6.8A$. Suppose 18-year Ben grows one inch over a 1-year period. Assuming his weight stays the same, which terms will be affected when calculating the change in his *BEE* during this time?

g. Without performing any calculations, explain how Ben's *BEE* will change.

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h. Examine the two *BEE* formulas again and explain what happens to a person's energy needs as they get older. Assume the person's weight stays constant.

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i. If your friend loses 20 pounds over the next 3 months, then what will the *BEE* indicate about his/her future energy needs?

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j. The actual amount of calories a person needs depends on the *BEE* and an activity factor. Since people expend a certain amount of energy in their normal daily activities, you need to increase the *BEE* according to your activity level. For example, a person who is sedentary needs 30% more calories. Moderate activity needs 40% more, and strenuous activity needs 70% more. Calculate your friend's energy needs for each activity level.

Sedentary =

Moderate Activity =

Strenuous Activity =

Feedback

The first step before applying either *BEE* formula is to convert an individual's weight and height to the metric units of kilograms and centimeters. For weight, the conversion factor $1\text{ kg} = 2.2\text{ lbs}$, can be written as a ratio equal to 1 in the following two ways:

$$\frac{2.2\text{ lb}}{1\text{ kg}} = 1 \quad \text{or} \quad \frac{1\text{ kg}}{2.2\text{ lb}} = 1$$

Suppose James is a 43-year old man with height 5'11" and weight 180 pounds. To convert his weight to metric units, multiply 180 pounds by the ratio that will cancel pounds and leave kilograms.

$$\begin{aligned} 180\text{ lb} \cdot \frac{1\text{ kg}}{2.2\text{ lb}} &= \frac{180\ \cancel{\text{lb}}}{1} \cdot \frac{1\text{ kg}}{2.2\ \cancel{\text{lb}}} \\ &= \frac{180\text{ kg}}{2.2} \\ &\approx 81.82\text{ kg} \end{aligned}$$

Another way you can convert pounds to kilograms involves solving a proportion. For example, letting x be the number of kilograms in 180 pounds, we have

$$\frac{2.2\text{ lb}}{1\text{ kg}} = \frac{180\text{ lb}}{x\text{ kg}}$$

$2.2 \cdot x = 1 \cdot 180$ Cross products must equal for proportion to be true.

$\dfrac{\cancel{2.2}\ x}{\cancel{2.2}} = \dfrac{180}{2.2}$ Divide both sides by 2.2 to solve for x.

$$x \approx 81.82\text{ kg}$$

For height, you first need to express 5'11" in terms of inches only. Since $1\text{ ft} = 12\text{ in.}$, we have

$$5\text{ ft} = 5 \cdot (1\text{ ft}) = 5 \cdot (12\text{ in.}) = 60\text{ in.}$$

So,

$$5\text{ ft } 11\text{ in.} = 60\text{ in.} + 11\text{ in.} = 71\text{ in.}$$

To convert inches to centimeters, the conversion factor, $1\text{ in.} = 2.54\text{ cm}$, can be written as a ratio equal to 1 in the following two ways:

$$\frac{1\text{ in.}}{2.54\text{ cm}} = 1 \quad \text{Or} \quad \frac{2.54\text{ cm}}{1\text{ in.}}$$

To convert James's height to metric units, multiply 71 inches by the ratio that will cancel inches and leave centimeters.

$$\begin{aligned}(71 \text{ in.}) \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} &= \frac{71 \text{ } \cancel{\text{in.}}}{1} \cdot \frac{2.54 \text{ cm}}{1 \text{ } \cancel{\text{in.}}} \\ &= 71 \cdot 2.54 \text{ cm} \\ &= 180.34 \text{ cm}\end{aligned}$$

With weight $W = 82.73$ kg, height $H = 180.34$ cm, and age $A = 43$ years, we can substitute these values into the *BEE* formula.

$$\begin{aligned}BEE &= 66 + (13.7 \cdot W) + (5 \cdot H) - (6.8 \cdot A) \\ BEE &= 66 + (13.7 \cdot 81.82) + (5 \cdot 180.34) - (6.8 \cdot 43) \\ BEE &= 66 + 1120.934 + 901.7 - 292.4 \\ BEE &\approx 1796 \text{ calories}\end{aligned}$$

Since the second ($13.7W$) and third ($5H$) terms are being added to the total, any increase in a person's weight or height will increase the total calories that person needs. Similarly, a decrease in weight or height will decrease the calories. The last term ($6.8A$) is subtracted from the total. This way, every time you have a birthday your *BEE* will decrease by 6.8 calories.

If James is moderately active, then you could calculate his caloric needs using either one of the following methods:

$$\begin{aligned}BEE + 40\% \text{ of } BEE &= 1796 + 0.40 \cdot 1796 \\ &= 1796 + 718.4 \\ &= 2514.4 \text{ calories}\end{aligned} \qquad \text{Or} \qquad \begin{aligned}140\% \text{ of } BEE &= 1.40 \cdot 1796 \\ &= 2514.4 \text{ calories}\end{aligned}$$

So James needs about 2514 calories per day based on his weight, height, and age. James knows that he has no control over his height and age but wonders whether his weight is at a healthy amount.

In the next Discovery Adventure, you will investigate how to find a person's *ideal body weight* using a rule. You will construct a table, look for patterns in the numbers, and develop a symbolic rule (or equation) to model the situation.

Discovery Adventure 3: Ideal Body Weight

There are tables available which claim to give a person's ideal body weight based on an individual's height. If you do not have access to these height/weight tables, then you can use the following rules to estimate ideal body weight.

Men – Use 106 pounds for the first 5 feet of height, plus 6 pounds for each inch over 5 feet.

Women – Use 100 pounds for the first 5 feet of height, plus 5 pounds for each inch over 5 feet.

a. Use the appropriate rule to estimate your friend's ideal body weight.

b. Complete the following table for women between 5 feet and 5 feet 6 inches. Use the given rule to set up your calculation. Leave the last row, second column blank for now.

Height (feet and inches)	Rule of Thumb	Weight (pounds)
5' 0"	$100 + 5 \cdot 0$	100
5' 1"	$100 + 5 \cdot 1$	105
5' 2"	$100 + 5 \cdot 2$	
5' 3"		
5' 4"		
5' 5"		
5' 6"		
⋮	⋮	
5' n"		

Note: The 3 vertical dots in the second to last row mean that the above pattern continues on.

c. Observe the pattern in the **sequence** of calculations in the second column. Note the values that remain constant and the values that change. If n represents the number of inches over 5 feet, then use a variable expression to represent the rule for finding an ideal body weight for women. Place your result in the last row, second column and after the equal sign below.

Ideal Body Weight =

d. Use another variable expression to represent the rule for finding an ideal body weight for men. Again let n represent the number of inches over 5 feet.

e. Do you think that these rules can determine ideal body weight for everyone? If yes, tell why. Otherwise, explain why certain individuals are exceptions to the rule.

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f. Percent ideal body weight is the ratio of your current weight divided by your ideal body weight,

$$\text{Percent Ideal Body Weight} = \frac{\text{Current Weight}}{\text{Ideal Body Weight}} \cdot 100$$

Calculate your friend's percent ideal body weight above and explain what this value means in terms of having a healthy weight.

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g. Assuming that the formula in part *f* gives accurate information, describe what a percentage over 100 would mean. Then describe what a percentage under 100 means.

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Feedback

The rules for ideal body weight can be expressed as a formula or equation by letting n be the number of inches over 5 feet.

$$\text{Women's ideal body weight} = 100 + 5n$$
$$\text{Men's ideal body weight} = 106 + 6n$$

If your current weight is the same as your ideal body weight, then the percent ideal body weight will be 100%. Therefore, any percent greater than 100% represent the part of your weight that is more than your ideal weight. For example, if an individual's percent ideal body weight is 110%, then that person has 10% more body weight than the ideal amount. Similarly, any percent under 100% means the part of your weight that is under the ideal. For example, if an individual's percent ideal body weight is 90%, then that person has 10% less body weight than the ideal amount.